# The Risk Premium of Corporate Bonds

*Non-monotonically related to credit ratings.*

Alexander Kozhemiakin

When do corporate bonds, the major subset of fixed-income securities subject to default risk, make a good investment? Any reasonable answer to this question depends on the expected risk premiums of corporate bonds or the extra compensation investors expect to receive for holding them instead of U.S. Treasury bonds with the same duration. The credit risk premium is similar to the much studied and debated equity risk premium, so critical for the valuation of stocks that it has been called "the financial equivalent of a cosmological concept" by a leading market practitioner.[1]

The expected equity risk premium is commonly defined as the extra return equityholders expect to achieve over the return on a relatively risk-free asset. To ensure a proper comparison with the equity risk premium, we should adjust the credit risk premium for the difference between the expected returns on the appropriate Treasury bonds (which have some price volatility) and shorter-dated T-bills, the standard risk-free benchmark. In general, for risk-averse investors, a positive expected risk premium should be a *sine qua non* in stock as well as bond portfolios.

What then is the current risk premium of corporate bonds? What has it been historically? Is it uniform across all credit ratings?

Even seasoned fixed-income professionals would find it difficult to answer these questions quite precisely. As is the case with its equity counterpart, not exactly knowing investor expectations at any given time greatly complicates the task of measuring the ex ante risk premium of corporate bonds. Although yield spreads of risky bonds over Treasuries are readily observable, it is hard to tell how much of the spread is simply a compensation for

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the expected default loss and how much of it is due to risk aversion or other factors.

I bring together two different insights to propose that, on average, the expected credit risk premium, defined as the non-default component of corporate bond spreads, is non-monotonically related to credit ratings. Over the course of a complete credit cycle, the risk premium is positive and high relative to actual spreads for investment-grade bonds. It rises in absolute terms as ratings deteriorate, reaching a peak in a BB sector, and then starts falling and becomes negative for CCCs (see Exhibit 1).[2]

I also suggest several potential reasons for such a peculiar shape of the expected risk premium curve and discuss its implications for portfolio strategies.

## CREDIT SPREAD PUZZLE

The first insight is an output of the rapidly growing group of bond pricing models explicitly estimating real-world or physical default probabilities. It is important to underline the difference between the physical probabilities of default and the more commonly used risk-neutral probabilities implied by corporate bond prices under a heroic assumption that the credit risk premium is zero. While risk-neutral probabilities, by definition, are worthless for our purposes, a good model estimating physical probabilities of default can mitigate the problem of unobservable expected default rates, thus helping to isolate the ex ante credit risk premium.[3]

There are two classes of pricing models that estimate default rates without requiring an assumption that the credit risk premium is zero. The first class falls under the general heading of reduced-form models—models that describe the evolution of corporate bond prices without reference to an underlying firm value process. Unlike most reduced-form models that deal with risk-neutral probabilities of default (e.g., Longstaff, Mithal, and Neis [2005]), the version of interest to me teases out physical default rates by relying on historical rating transition matrices provided by the major rating agencies or by relying simply on average historical default probabilities in each credit rating category (Elton et al. [2001], Amato and Remolona [2005]).

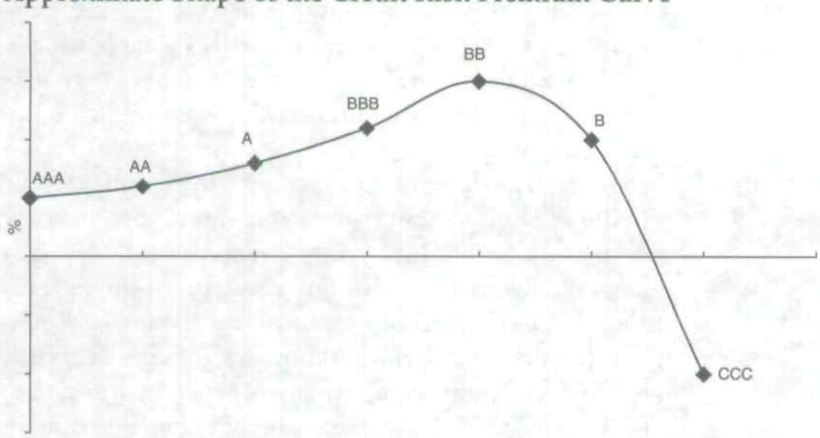
The second class of models is represented by a more mathematically complex structural approach that is based on the option pricing methodology pioneered by Black and Scholes [1973]. This approach views the variability of the firm's asset value as driving the default process (Merton [1974], Huang and Huang [2002]). According to structural models, a firm defaults when the market value of its assets falls below a certain threshold. For example, according to Moody's KMV EDF model, the most commercially successful application of the structural approach, this threshold is the firm's liabilities payable.

Although the two classes of models are quite different in their methodologies, they tend to reach a similar conclusion that spreads on corporate bonds are much wider than what would be required to compensate for expected credit losses alone. This conclusion, my first insight, has come to be known as the *credit spread puzzle*.[4] While the extent of the difference between actual spreads and theoretical predictions has received the most attention, bond pricing models also generate counter-factually low predictions for the time variation in credit spreads (see Chen, Collin-Dufresne, and Goldstein [2005]).

I assume here that the difference between observed bond prices and those predicted on the basis of expected default losses is due to the credit risk premium. This is, of course, an oversimplification, as there are other factors influencing the pricing of corporate bonds, such as higher taxes and lower liquidity relative to U.S. Treasury issues. The illiquidity of corporate bonds is particularly often cited as a major reason for the positive non-default components of spreads. In fact, some practitioners refer to the non-default component of corporate bond spreads as the illiquidity spread (e.g., Rappoport [1999]).

**EXHIBIT 1**

**Approximate Shape of the Credit Risk Premium Curve**



Yet the non-default component of spreads is simply too large to be explained solely by corporate illiquidity. Indeed, it is reasonable to suggest that the portion of the spread due to diminished liquidity (captured by bid-ask spreads) is similar for corporate bonds rated in adjacent categories, say, BBB and A. Then the BBB-A rating spread should be primarily due to a marginal increase in expected losses.

Actual rating spreads are much wider than the compensation for an increase in expected credit losses calculated on the basis of structural and rating transition models, however. The discrepancy implies sizable incremental credit risk premiums.

Unfortunately, estimating the required compensation for the relative lack of liquidity is not an easy task. As pointed out by Amato and Remolona [2005], compensation for illiquidity ultimately depends on the nature of a marginal investor in corporate bonds. If the marginal investor is a buy-and-hold institution, there should not be much compensation for illiquidity. The reverse is true if a marginal investor is a hedge fund involved in active trading.

In the final analysis, the liquidity premium and the credit risk premium are closely related. Both concepts focus on the need for extra compensation for uncertainty about future price levels. This uncertainty may stem from the possibility of unexpected gains and losses or changes in liquidity conditions. Separating the two might be useful, but that question is beyond the scope of this study.[5]

The credit risk premium generally rises in absolute terms as ratings deteriorate. Most published models focus on investment-grade bonds, presumably because of data constraints. Models that include speculative-grade issues suggest that the credit risk premium continues to rise for less creditworthy bonds, at least until the BB rating. If expressed as a percentage of actual spreads, however, the credit risk premium falls with a decline in credit quality. For top-rated bonds, virtually all the actual spread is due to the non-default component.

Exhibit 2 describes the differences between actual and predicted spreads for two representative and publicly available models. The first model is of the ratings transition (reduced-form) type, and the second is structural—for bonds with five- to seven-year and four-year maturities, respectively.

While the results in Exhibit 2 are illustrative, we should be careful not to read too much into the exact numbers, as they can change depending on model specification, time period, and bond duration.

Furthermore, each type of model has its weaknesses. Structural models rely too much on the equity market, and can be of questionable accuracy (see Eom, Helwege, and Huang [2004]). Because the market value of assets is not directly observable, structural models treat the firm's equity value as a call option on the firm's underlying assets. Hence, default probabilities produced by structural models are basically determined by a company's stock price combined with its latest financial statements. Consequently, all else equal, a drop in the stock price would produce a higher estimated default probability, even though the stock move might be the result of some exogenously driven spike in risk aversion and not a change in the company's actual creditworthiness. Thus, even the structural model may not be able to isolate a pure physical probability of default.

Although we can derive default probabilities from structural models without making any assumptions about the credit risk premium, the estimates will still be contaminated by changes in the equity risk premium. This becomes

## EXHIBIT 2
**Non-Default Component of Corporate Spreads by Rating**

| Ratings | **Rating-transition model** (for 5-7 year maturity) | | | | **Structural model** (for 4 year maturity) | | | |
|---|---|---|---|---|---|---|---|---|
| | Average spreads (1997-2004) | | Difference | Difference | Average spreads (1973-1998) | | Difference | Difference |
| | Predicted (bps) | Actual (bps) | (bps) | (% of actual) | Predicted (bps) | Actual (bps) | (bps) | (% of actual) |
| AAA | 0 | 70 | 70 | 100 | 1 | 55 | 54 | 98 |
| AA | 1 | 81 | 80 | 99 | 6 | 65 | 59 | 91 |
| A | 6 | 108 | 102 | 94 | 10 | 96 | 86 | 90 |
| BBB | 40 | 182 | 141 | 78 | 32 | 158 | 126 | 80 |
| BB | 148 | 339 | 191 | 56 | 172 | 320 | 148 | 46 |
| B | n/a | n/a | n/a | n/a | 446 | 470 | 24 | 5 |

*Sources: Amato and Remolona [2005], Huang and Huang [2002], author's calculations.*

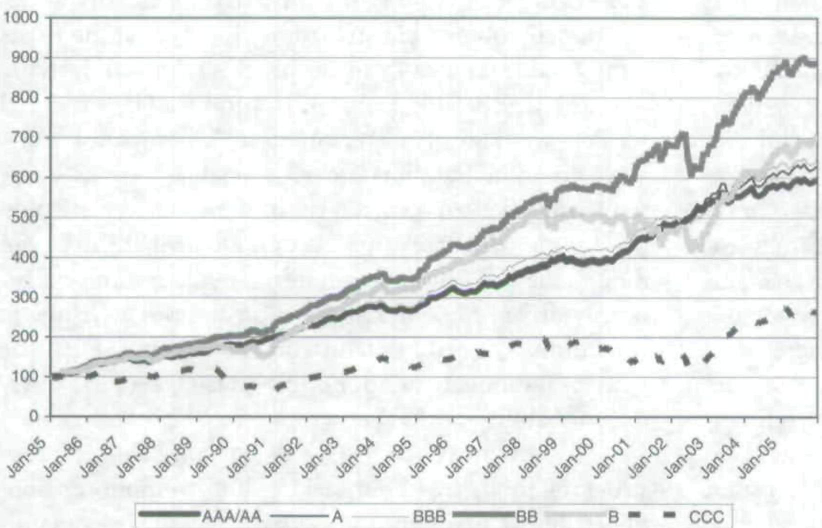
# EXHIBIT 3

**Standard & Poor's One-, Five-, and Ten-Year Cumulative Issuer-Weighted Default Rates (%) (1981–2005)**

| | One-year default rate | | Five-year default rate | | Ten-year default rate | |
|---|---|---|---|---|---|---|
| | Average | Standard deviation | Cumulative average | Standard deviation | Cumulative average | Standard deviation |
| AAA | 0 | 0 | 0.1 | 0.22 | 0.44 | 0.83 |
| AA | 0.01 | 0.04 | 0.29 | 0.38 | 0.81 | 0.78 |
| A | 0.04 | 0.07 | 0.59 | 0.4 | 1.83 | 0.93 |
| BBB | 0.27 | 0.29 | 2.83 | 1.3 | 5.82 | 1.8 |
| BB | 1.12 | 1.12 | 10.65 | 4.32 | 18.29 | 4.46 |
| B | 5.38 | 3.09 | 24.16 | 7.57 | 32.38 | 4.32 |
| CCC/C | 27.02 | 12.54 | 47.56 | 14.45 | 53.05 | 10.37 |

*Source: Standard & Poor's.*

# EXHIBIT 4




*Source: Citigroup.*

even more of a problem if the equity risk premium and the credit risk premium move in tandem.

The major shortcoming of rating transition models is that the variance of rating transition matrices is fairly high and that default rates can fluctuate quite significantly from their long-term averages, especially for lower categories of ratings (see Exhibit 3). The major rating agencies appear to use a through-the-cycle approach, and adjust credit ratings only in response to changes in long-term prospects (Altman and Rijken [2004]). Therefore bond ratings are more stable than a firm's underlying creditworthiness.

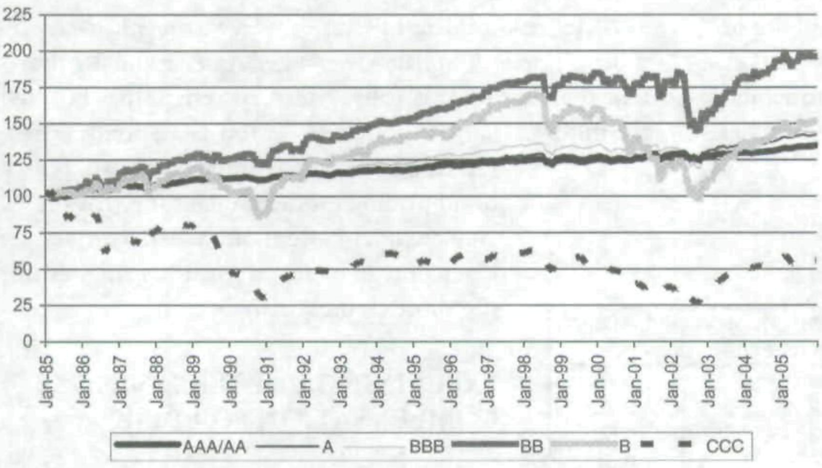
Indeed, at least for Moody's, the more nimble conditional ratings incorporating an issuer's rating outlook (positive, negative, stable) and rating review status (review for upgrade or downgrade) do a better job of estimating the probability of default over one- and three-year horizons than the standard ratings, which are less prone to change (Hamilton and Cantor [2004]). As a result, until more comprehensive tests are conducted, it would be prudent to be cautious about the ability of rating transition models to produce reliable out-of-sample estimates of physical default rates.

As a matter of fact, assuming that investors are generally risk-averse, the volatility of default rates for lower-rated bonds suggests that relatively high risk premiums are a puzzle for only top-rated investment-grade bonds (especially those with short maturities) whose historical default rates have been both extremely low and non-volatile. In quite large investment-grade portfolios, diversification should be able to prevent actual losses from exceeding expected losses by an amount significant enough to warrant a high ex ante risk premium.

For speculative-grade issues, however, default rates and ultimately returns are much less predictable. In such more risky portfolios, requiring compensation for potentially sizable unexpected losses (whether from actual defaults or from repricing of the credit risk premium itself) is not puzzling at all. This extra compensation for an undesirably high level of uncertainty is quite consistent with the positive equity risk premium that stock market participants have historically expected to earn, according to the available survey data.[6]

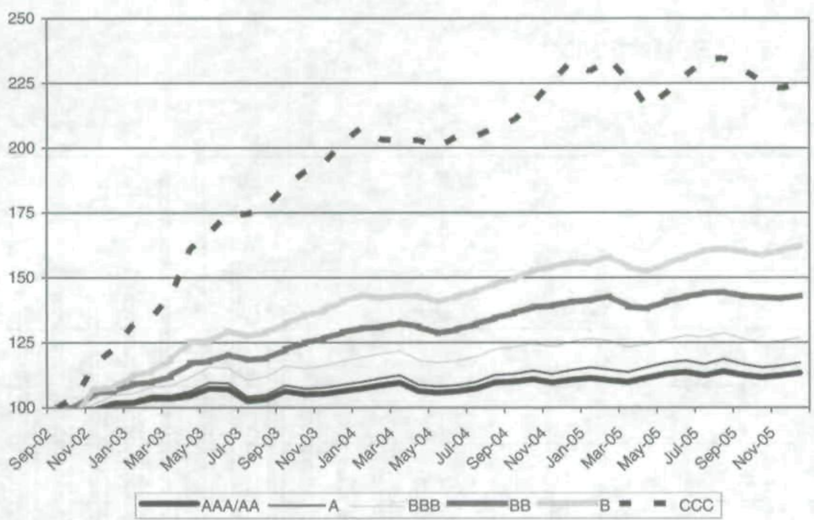
**EXHIBIT 5**

**Cumulative Excess Returns by Rating (January 1985 = 100)**




*Source: Citigroup, Bloomberg, author's calculations.*

**EXHIBIT 6**




*Source: Citigroup.*

## PEAK IN THE REALIZED RISK PREMIUM

An interesting result of the structural approach reported by Huang and Huang [2002] and presented in Exhibit 2 is that the expected credit risk premium peaks in absolute terms for BBs, after which it falls quite dramatically for Bs. Although published bond pricing models estimating physical default probabilities do not typically focus on the lower ratings, many practitioners have long noticed that it is not higher-yielding Bs or CCCs—bonds subject to the highest risk of default—but BBs that tend to outperform other credit tiers on a total return basis.[7] This observation is even more puzzling than the credit spread puzzle, as it means a breakdown of the familiar risk-return trade-off.

The second insight—the historical outperformance of BBs—implies that BBs have had the highest credit risk premium of all other rating categories on an ex post facto basis. It is therefore quite consistent with a finding by Huang and Huang [2002] that the ex ante risk premium peaks at BB. Obviously, however, expected and realized risk premiums do not necessarily have to match, as investor expectations of future returns can be off the mark.

Exhibit 4 plots cumulative total returns for each major rating category of corporate bonds since 1985.[8] From the highest-rated credits until BBs, more risky bonds have outperformed less risky ones on an absolute basis. After BBs, however, the relationship between the level of credit risk and historical returns is reversed. Bs underperform BBs, especially during the two recessions (1990-1991, 2001) and CCCs register the worst returns. In fact, the performance of the CCC sector has been so poor over this period that it has even failed to generate positive excess returns over Treasury bonds with comparable duration (see Exhibit 5). In other words, after reaching its peak for the BBs, the realized risk premium declines as ratings deteriorate, and becomes negative for CCCs.

This is not to argue that CCCs, on average, have always been an inferior investment. In a recovery phase following a recession, CCCs can perform quite strongly. With the benefit of hindsight, Exhibit 6 presents the cumulative returns for the major rating categories of corporate bonds since the very bottom of the latest credit cycle (September 2002). In this very supportive period for corporate credit, CCCs have outperformed other ratings categories. More generally, lower-rated issues have acted in a manner consistent with their perceived high-beta status, outperforming better-rated bonds in the rising market.

Good investors can therefore add value to their corporate bond portfolios by tactically adding Bs and CCCs during certain market upswings and removing them early enough to avoid large losses. Of course, successful implementation of this strategy requires considerable market-timing skill. For buy-and-hold investors looking to achieve the highest returns, BB-rated bonds are still the best bet, as the historical evidence suggests that on a long-term, through-the-cycle, basis they have had the best absolute performance.

In risk-adjusted terms, however, the most creditworthy bonds dominate lower-rated issues. Exhibit 7 shows that the very low volatility of excess returns has boosted the historical information ratios for top-rated bonds. These results go hand in hand with the output of bond pricing models, suggesting that the ex ante credit risk premium, when expressed as a percentage of actual spreads, is highest for the most creditworthy bonds.

EXHIBIT 7

**Historical Returns and Risk (1985-2005) (%)**

| | Annualized return | Annualized excess return | Annualized std. deviation of excess returns | Information ratio |
|---|---|---|---|---|
| AAA/AA | 8.9 | 1.4 | 1.9 | 0.76 |
| A | 9.2 | 1.7 | 2.2 | 0.76 |
| BBB | 9.3 | 1.8 | 3.3 | 0.53 |
| BB | 11.0 | 3.3 | 6.5 | 0.51 |
| B | 9.7 | 2.0 | 9.0 | 0.23 |
| CCC | 4.8 | -2.7 | 15.2 | -0.18 |

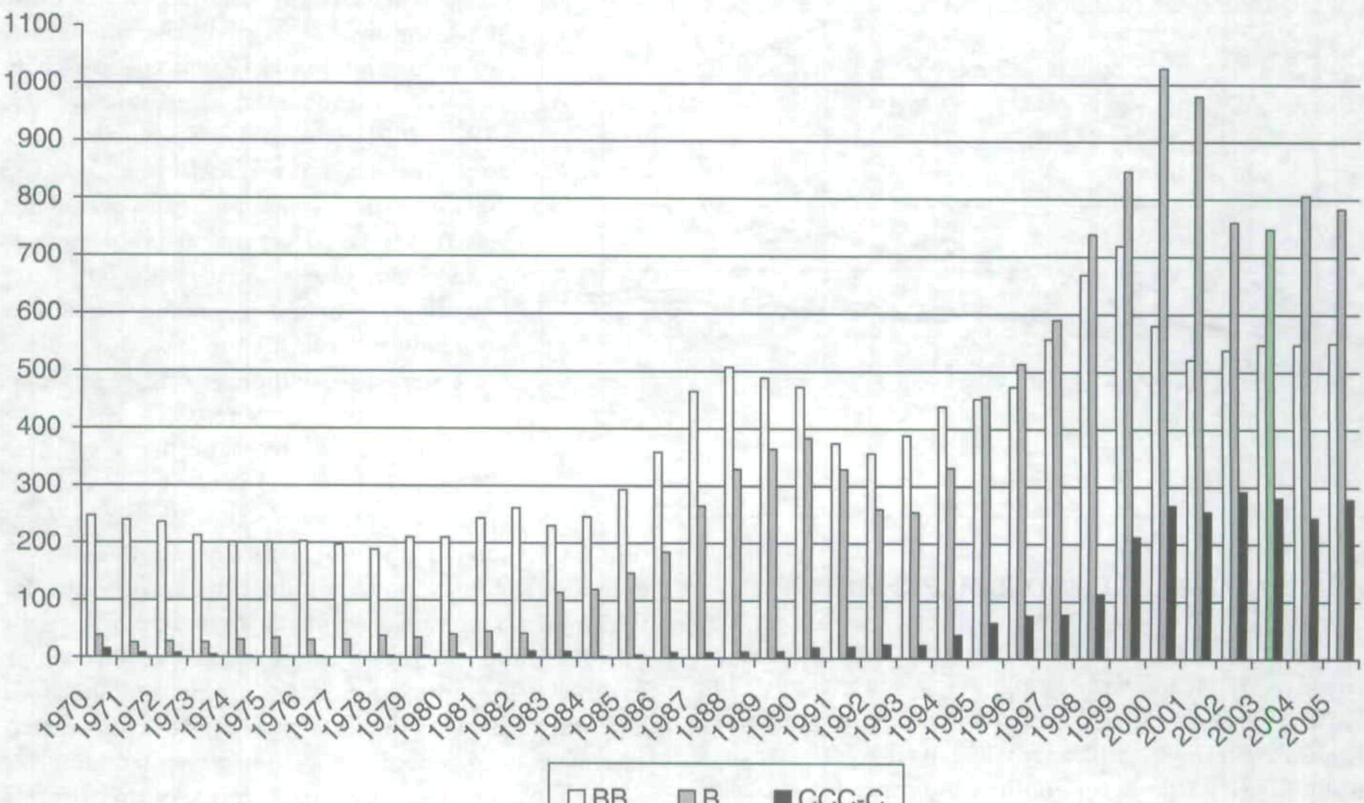
*Source: Citigroup, Bloomberg, author's calculations.*

## EXPLAINING THE DECLINING COMPENSATION FOR RISK

Why do the realized risk premiums decline in absolute terms after BBs? The answer to this question appears to be twofold.

EXHIBIT 8

**Number of Issuers Rated BB, B, and CCC-C by Moody's (1970-2005)**



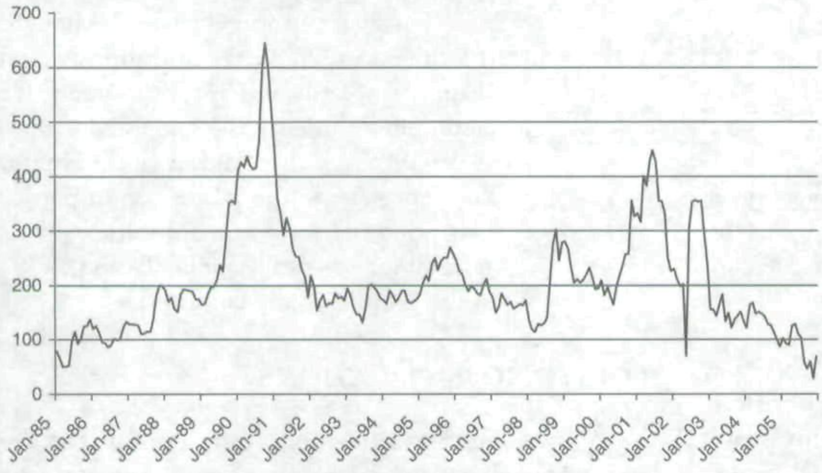

*Source: Moody's.*

## Exhibit 9
**Average Annual Issuer-Weighted Default Rates (%) (1985-2005)**

| | BB | | B | | CCC/C | |
|---|---|---|---|---|---|---|
| | No recession | Recession | No recession | Recession | No recession | Recession |
| Standard & Poor's | 0.8 | 2.8 | 4.6 | 11.2 | 22.8 | 36.6 |
| Moody's | 1 | 3.4 | 5.2 | 13.4 | 18.1 | 43.3 |

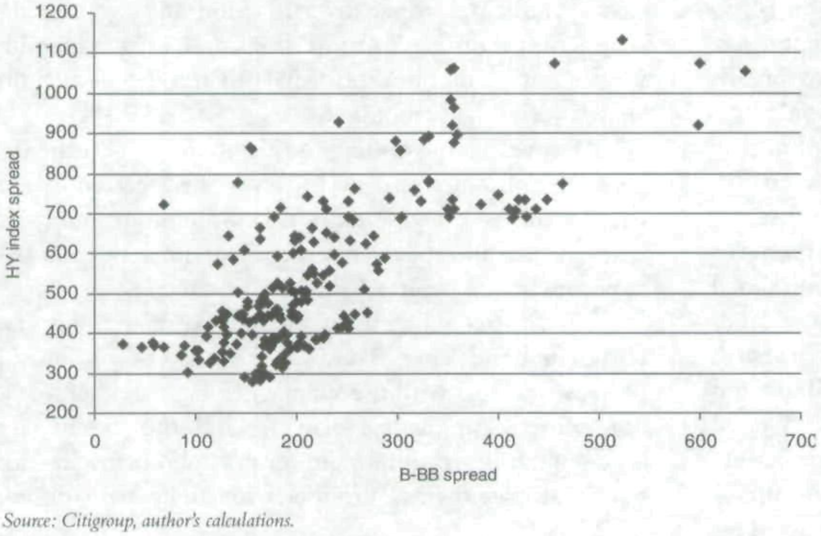
*Sources: Standard & Poor's, Moody's, author's calculations.*

## Exhibit 10
**Historical B-BB Spread Differential (bp)**




*Source: Citigroup, author's calculations.*

## Exhibit 11
**B-BB Spread Differential versus HY Index Spreads (bp)**




*Source: Citigroup, author's calculations.*

It might be argued that the observed drop in the realized risk premiums for the least creditworthy bonds was not intentional and that the ex ante risk premium for Bs and CCCs was indeed high. Perhaps, given a relatively short history of Bs and CCCs, investors have simply underestimated their propensity to default. Indeed, while bonds rated BB and better have had a much longer trading and default history, it was only in the early 1980s and late 1990s (respectively) that there were as many as 100 issuers rated B or CCC by the major rating agencies (see Exhibit 8).

Moreover, for any given rating, speculative-grade default rates spike quite dramatically in recessions (see Exhibit 9). A recession depresses, and an expansion raises, revenues of levered corporations. Yet for many corporations operating and debt servicing costs remain fixed. As a result, annual default rates fluctuate with the macroeconomic cycle (Kozhemiakin [2005]). Consequently, as there have only been two recessions in the past 20 years, investors have experienced just two historical stress cases for the B category (1990-1991 and 2001) and only one for CCCs (2001).

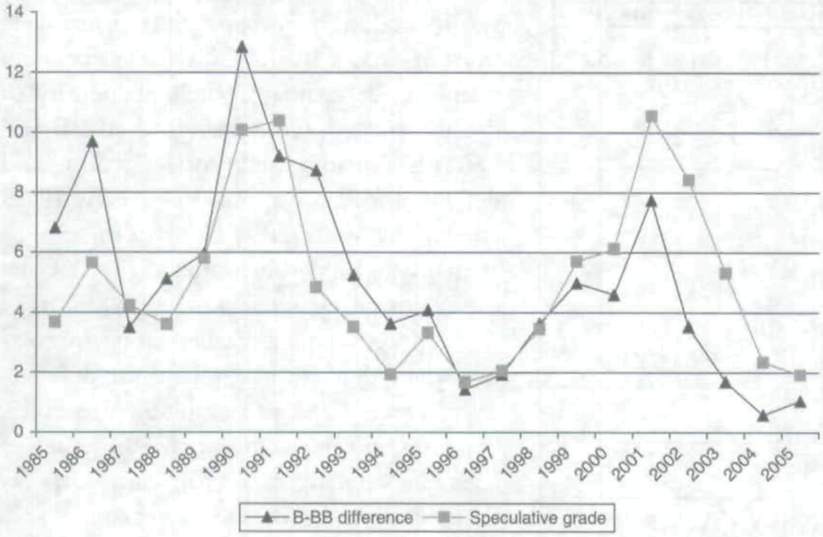
Nevertheless, it appears that if there were indeed a genuine error in risk assessment, investors by now would have most likely learned from their experience. At the very least, one could reasonably expect Bs to be more cautiously priced relative to BBs than has been the case in the past two decades. Yet the tightness of the current B-BB spread differentials suggests this is not the case (see Exhibit 10).

To be fair, it should be noted that the B-BB spread is market directional, tightening when the high-yield market is strong and widening during sell-offs (see Exhibit 11). This procyclical relationship is quite reasonable from a fundamental standpoint, as annual default rates of B-rated bonds come closer to the levels of BB default rates at the peaks of the credit cycle (see Exhibit 12).

As aggregate speculative-grade default rates are now at their historical lows, it is therefore not surprising to see a reduction in the B-BB spread differential. Yet it is the

## EXHIBIT 12

**Difference Between Annual Default Rates of B and BB-Rated Issuers versus Annual Default Rates for Entire Speculative-Grade Sector (%)**




*Source: Moody's, author's calculations.*

*level* of the B-BB spread (only 68 basis points as of the end of 2005) that appears too low, given the historical record of default losses—unless, of course, investors expect the present extremely supportive general credit environment to persist for the next few years.[9]

Alternatively, we can accept the premise that the realized risk premiums for speculative-grade issues have been more or less an accurate representation of the ex ante risk premiums, if not in cardinal terms then at least on the ordinal scale. So why would the expected returns on Bs and especially CCCs be lower than the returns on BBs?

The answer seems to have at least three components. The first has to do with why the expected returns for BBs have been so high. The most plausible explanation points to inefficiencies due to market segmentation and in particular to the lack of natural buyers for bonds rated BB, which may be called an orphan of the credit universe.

That is, on the one hand, BBs are out of reach for investors (e.g., insurance companies) who are constrained from holding subinvestment-grade bonds. On the other hand, portfolio managers selling a product to retail investors by focusing primarily on its yield may be tempted to forgo BBs in favor of higher-yielding B- or CCC-rated bonds. The fallen angel effect, in which formerly investment-grade bonds enter the BB category at fairly attractive prices because they have to be sold by the accounts that can no longer hold them because of their speculative-grade rating, may have also contributed to the outperformance of BBs.

Second, taken as a group, high-yield investors may have overestimated their security selection skills. While portfolio managers might have agreed that B and CCC tiers were not attractively priced on average, the search for yield could have compelled them to buy individual B or CCC names (bidding up the whole sector in the process) in an erroneous belief that they would outperform their respective rating categories.

Finally, investors may have also bought the lowest credit tiers continuously, and again erroneously, anticipating a supportive credit environment. As I suggested earlier, at certain times the realized risk premium does not peak in the BB sector. In particular, good investors can add value to their portfolios by tactically adding Bs and CCCs at the bottom of the credit cycle.

### PORTFOLIO IMPLICATIONS

The emerging non-monotonic contour of the expected credit risk premium curve has important implications for corporate bond portfolio strategies. If investors are concerned primarily with generating the best risk-adjusted returns, the shape of the risk premium curve will promote investment-grade bonds. The credit risk premiums for bonds of the most creditworthy issuers are positive and high relative to the actual spreads. This also implies that 1) ensuring proper diversification and 2) reducing transaction costs are of more importance in managing investment-grade bond portfolios than a detailed credit analysis of individual bonds.

If investors are willing to tolerate more volatility to get better absolute returns, however, the location of the peak in the ex ante and ex post risk premiums indicates that crossing into speculative-grade territory to get a BB exposure is the preferred structural strategy.

Finally, according to two decades of trading history, corporate bonds rated B and lower should be considered only by investors with the superior ability to either select outperforming individual securities or to time the market accurately. It is very important for portfolio managers not to overestimate their security selection and market-timing skills.

## ENDNOTES

The author thanks Michael Atkin, Rob Bloemker, Jeffrey Kaufman, and David Waldman for insightful comments.

[1]Martin L. Leibowitz at the 2002 Equity Risk Premium Forum, AIMR.

[2]I use only the Standard & Poor's rating scheme in order to avoid excessive notation. Thus, Moody's Aaa rating is coded as AAA, Aa as AA, and so on.

[3]To simplify, recovery values are treated as constant. In reality, however, recovery values and aggregate default rates are positively correlated (Altman et al. [2006]).

[4]This is different from the widely cited equity premium puzzle, which refers to a large gap between various measures of the ex ante risk premium on the one hand and the ex post premium actually realized by stock investors on the other.

[5]An increasingly popular way to measure the liquidity premium is to compare spreads on corporate bonds and credit default swaps (CDS). According to this approach, the contractual nature of a CDS (in particular, unlimited notional amounts) makes its pricing less vulnerable to liquidity distortions. The greatest difficulty in estimating the liquidity premium based on CDS is that there are several other reasons why the default swap basis can fluctuate.

[6]See, for example, the Philadelphia Fed's Survey of Professional Forecasters for stock and T-bill return expectations.

[7]See, for example, Loyes and Fransolet [2004].

[8]The earliest period for which there are reliable trading data for speculative-grade issues.

[9]According to Citigroup (the source of spread data used in this study), the modified duration of B-and BB-rated bonds was approximately 5.3 years as of the end of 2005.

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